Interpretation of y-scaling of the nuclear response

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The behavior of the nuclear matter response in the region of large momentum transfer, in which plane wave impulse approximation predicts the onset of y-scaling, is discussed. The theoretical analysis shows that scaling violations produced by final state interactions are driven by the momentum dependence of the nucleon-nucleon scattering cross section. Their study may provide valuable information on possible modifications of nucleon-nucleon scattering in the nuclear medium.

Inclusive scattering of high energy electrons off nuclear targets has long been recognized as a powerful tool to measure the nucleon momentum and removal energy distribution [1]. The underlying picture is that at large momentum transfer electron-nucleus scattering reduces to the incoherent sum of elementary scattering processes involving individual nucleons, distributed in momentum and removal energy according to the spectral function $P(\mathbf{k}, E)$. In the early seventies, West first pointed out [2] that, if the electron-nucleon processes are elastic and the final state state interactions between the struck particle and the spectator system can be neglected, the nuclear response $S(\mathbf{q}, \omega)$, which generally depends upon both momentum ($\mathbf{q}$) and energy (ω) transfer, exhibit scaling, i.e. it can be simply related to a function of only one kinematical variable, denoted y. Within the simplest non-relativistic approximation, y can be identified with the minimum projection of the nucleon momentum along the direction of the momentum transfer, while the scaling function $F(y) = (q/m)S(\mathbf{q}, \omega)$, where m denotes the nucleon mass, can be directly written in terms of the nucleon momentum distribution.

The approach to y-scaling in few-nucleon systems [3,4] and medium-heavy nuclei [5] has been experimentally investigated at SLAC in the kinematical domain extending up to $Q^2 = q^2 - \omega^2 \sim 3$ (GeV/c)^2. Recent measurements carried out at TJNAF using Carbon, Iron and Gold targets have extended the Q^2 range up to ~ 7 (GeV/c)^2 [6]. The available data, plotted as a function of y, display a striking overall scaling behavior at $y < 0$, corresponding to $\omega < \omega_{QE}$ ($\omega_{QE} = Q^2/2m$, is the energy transfer associated with elastic scattering off a free nucleon at rest), indicating that elastic scattering off individual nucleons is indeed the dominant reaction mechanism in that region. However, the analysis of the Q^2-dependence at fixed y shows sizeable scaling violations at $y < -0.2$ GeV/c and $Q^2 < 3$ (GeV/c)^2, to be ascribed mainly to final state interactions (FSI). Recenlty, Donnelly and Sick [7] have also shown that the y-scaling functions of different nuclei exhibit scaling of the *second kind* in the new variable $\psi' = y/k_F$, k_F being the Fermi momentum.

The relevance of FSI in the kinematical regime of the SLAC data has been systematically studied in refs. [8–10], within a microscopic many-body approach in which nucleon-nucleon (NN) correlations are consistently taken into account in both the initial and final state. The results of these calculations confirm that FSI effects are important, particularly in the region of large negative y, and have to be included to quantitatively account for the measured cross sections.

The extension of the analysis of refs. [8–10] to the region $3 < Q^2 < 7$ (GeV/c)^2, relevant to the interpretation of the new TJNAF data, requires the full calculation of the nuclear cross section, including the electromagnetic vertex, and the use of spectral functions adequate to describe finite targets, that can be obtained within the local density approximation [10]. In this paper we avoid these complications and focus only on the mechanisms driving the approach to y-scaling of $S(\mathbf{q}, \omega)$ in the case of infinite nuclear matter, a system whose spectral function can be obtained from *ab initio* microscopic calculations using realistic nuclear hamiltonians [11].

The most popular argument supporting the expectation that FSI effects become negligibly small at large momentum transfer is based on the observation that, compared to the PWIA amplitude, the amplitude of the process including a rescattering in the final state involves an extra propagator, associated with the struck nucleon carrying a momentum $\sim \mathbf{q}$. As a consequence, this amplitude is expected to be suppressed when q is large. The validity of this argument has been proved in the context of a fully nonrelativistic model based on G-matrix perturbation theory, in which FSI were described in terms of elastic NN scattering processes [12]. Within this picture, the disappearance of FSI effects at large momentum transfer simply reflects the fact that the elastic NN scattering cross section is a rapidly decreasing function of q. However, when the momentum of the struck nucleon is in the 2–5 GeV/c range, typical of the TJNAF kinematical regime, inelastic scattering is known to become

dominant and must be taken into account. The inelastic cross section is roughly momentum independent, leading to a scattering amplitude that grows linearly with q. Therefore, in spite of the suppression coming from the nucleon propagators, inelastic rescatterings may give rise to FSI effects that remain nearly constant as the momentum transfer increases, thus preventing the onset of the y-scaling regime.

The treatment of FSI developed in ref. [8] is based on the high-energy approximation, i.e. on the assumptions that i) the fast struck nucleon moves along a straight trajectory with momentum $\sim \mathbf{q}$ (eikonal approximation) and ii) the spectator system can be seen as a collection of fixed scattering centers (frozen approximation). The resulting quasielastic response reads

$$S(\mathbf{q}, \omega) = \int d\omega' S_{PWIA}(\mathbf{q}, \omega') F_q(\omega - \omega') , \qquad (1)$$

where the PWIA response is given by

$$\begin{aligned} S_{PWIA}(\mathbf{q}, \omega) = \int d^3k \; dE \; P(\mathbf{k}, E) \\ \times \; \delta(\omega - E - \sqrt{|\mathbf{k} + \mathbf{q}|^2 + m^2} + m) , \quad (2) \end{aligned}$$

and the folding function $F_q(\omega)$ is defined as

$$F_q(\omega) = \int_{-\infty}^{+\infty} \frac{dt}{2\pi} \; e^{i\omega t} \; U_q(t) . \qquad (3)$$

The effects of FSI are described by the function $U_q(t)$, which can be written [13]

$$U_q(t) = \frac{1}{A} \left\langle \sum_{i=1}^A U_q^{(i)}(R, t) \right\rangle , \qquad (4)$$

where $\langle \ldots \rangle$ denotes the expectation value in the target ground state, $R \equiv \{\mathbf{r}_1, \mathbf{r}_2, .., \mathbf{r}_A\}$ specifies the target configuration and

$$\begin{aligned} U_q^{(i)}(R, z) = 1 + \left(\frac{i}{q}\right) \int_0^z dz_1 \sum_{j \neq i} \Gamma_q(|\mathbf{r}_i + \hat{q}z_1 - \mathbf{r}_j|) \\ + \frac{1}{2} \left(\frac{i}{q}\right)^2 \int_0^z dz_1 \sum_{j \neq i} \Gamma_q(|\mathbf{r}_i + \hat{q}z_1 - \mathbf{r}_j|) \\ \times \int_0^z dz_2 \sum_{k \neq j \neq i} \Gamma_q(|\mathbf{r}_i + \hat{q}z_2 - \mathbf{r}_k|) + \ldots . \quad (5) \end{aligned}$$

In the above equation $\hat{q} = (\mathbf{q}/q)$, while $z = vt$ is the distance travelled by the struck particle during a time t past the electromagnetic interaction. The dynamics of the rescattering process is dictated by the function $\Gamma_q(r)$, simply related to the amplitude for NN scattering at incident momentum q and momentum transfer $\mathbf{k}'$, $f_q(\mathbf{k}')$, through

$$\Gamma_q(r) = \int \frac{d^3k'}{(2\pi)^2} \; f_q(\mathbf{k}') \; e^{i\mathbf{k}' \cdot \mathbf{r}} . \qquad (6)$$

Eqs.(1)-(6) clearly show that in absence of FSI $U_q(t) \equiv 1$, implying in turn $F_q(\omega) = \delta(\omega)$, and the PWIA result is recovered. From eq.(5) it is also apparent that, as expected, processes involving n rescatterings exhibit a $(1/q)^n$ dependence associated with the nucleon propagators. However, the presence of the $(1/q)^n$ factors does not guarantee that FSI corrections become vanishingly small at large q, since $U_q^{(i)}(R, z)$ has an additional q-dependence coming from the scattering amplitude $f_q(\mathbf{k}')$.

According to the optical theorem, the imaginary part of the forward amplitude, which is known to be dominant at high q, can be written in terms of the total scattering cross section $\sigma_{tot}(q)$ as

$$Im \; f_q(0) = \frac{q}{4\pi} \; \sigma_{tot}(q) . \qquad (7)$$

Eqs.(5)-(7) show that the $(1/q)^n$ factors appearing in the n-th rescattering contribution to $U_q^{(i)}(R, z)$ cancel, and the q-dependence of the FSI corrections is eventually driven by $\sigma_{tot}(q)$.

The measured NN total cross section is roughly constant in the momentum range 2–5 GeV/c [14]. Hence, in this region the $F_q(\omega)$ obtained from eqs.(3)-(5) using the free-space $\sigma_{tot}(q)$ does not show any appreciable q-dependence. This feature is illustrated in fig. 1, where the folding functions evaluated in infinite nuclear matter at $q = 2.2$ and 3.4 GeV/c are compared.
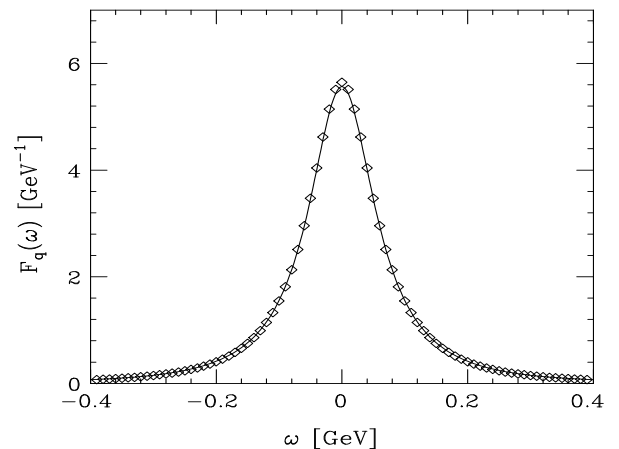

FIG. 1. Folding functions in infinite nuclear matter, evaluated using eqs.(3)-(6) and the parametrization of the NN scattering amplitude of refs. [15,16]. The solid line and the diamonds correspond to $q = 2.2$ and 3.4 GeV/c, respectively.

At large q, the shape of the PWIA response, defined as in eq.(2), also becomes nearly independent of q, as shown in fig. 2. The nuclear matter $S_{PWIA}(\mathbf{q}, \omega)$ exhibits a bump whose maximum, corresponding to $\omega \sim \omega_{QE} = \sqrt{q^2 + m^2} - m$, moves towards higher values of ω as q increases, whereas its height and width remain almost constant, being dictated by the Fermi momentum k_F. Note that this feature is to be ascribed to the use of relativistic kinematics in the energy-conserving δ-function

of eq.(2). In the nonrelativistic regime the width of the bump in the quasielastic response increases linearly with q, while its height displays a $(1/q)$ behavior.
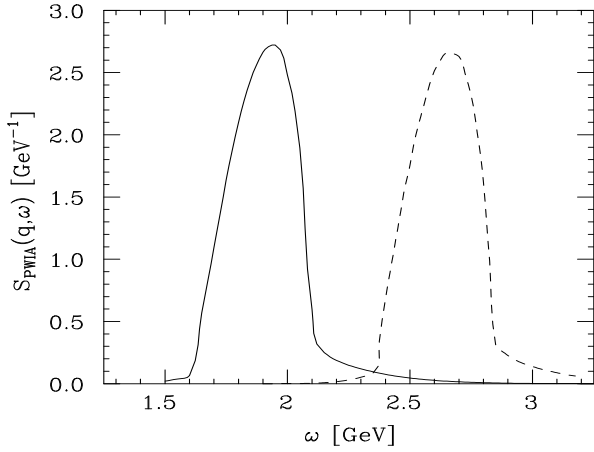

FIG. 2. Nuclear matter response functions evaluated within PWIA using eq.(2) and the spectral function of ref. [11]. The solid and dashed lines correspond to $q = 2.6$ and 3.4 GeV/c, respectively.

When relativistic kinematics is used, the y-scaling variable for infinite nuclear matter is defined as

$$y = -q + \sqrt{(\omega - E_{min})^2 + 2m(\omega - E_{min})} \,, \qquad (8)$$

E_{min} being the minimum energy needed to remove a nucleon from the nuclear matter ground state. The corresponding scaling function reads

$$F(q,y) = \frac{q}{\sqrt{m^2 + (y+q)^2}} \, S(\mathbf{q}, \omega). \qquad (9)$$

From the above equations, it can be readily seen that, if the folding function $F_q(\omega)$ and the shape of the PWIA response become q-independent at large q, $F(q,y)$ obtained using $S(\mathbf{q}, \omega)$ of eq.(1) is q-independent as well. Hence, in this case $F(q,y)$ *does* exhibit scaling but its value as $q \to \infty$ *does not* correspond to the PWIA limit, and cannot be simply related to the nuclear spectral function. In fig. 3 the PWIA scaling function obtained using the spectral function of ref. [11], (dot-dash line) is compared to the result of the calculation including FSI, carried out with the parametrization of the NN scattering amplitude of refs. [15,16] (solid line labelled σ_{tot}). Note that the PWIA $F(q,y)$ approaches y-scaling from below, whereas the occurrence os FSI leads to a convergence from above. The solid line labelled σ_{el} shows the results obtained neglecting the contribution of inelastic processes, i.e. replacing the total cross section with the elastic cross section in the definition of the NN scattering amplitude (see eq.(7)). The different behavior of the two solid lines reflects the fact that while σ_{tot} changes by less than 10 % over the range $1 < q < 4$ GeV/c, the ratio σ_{el}/σ_{tot} drops from ~ 95 % to ~ 30 %.

As pointed out in ref. [8], using the free-space amplitude to describe NN scattering in the nuclear medium may be questionable. Pauli blocking and dispersive corrections are known to be important at moderate energies [17]. However, their effects on the calculated nuclear response have been found to be small in the kinematics of the SLAC data, corresponding to $q \sim 2$ GeV/c, and decrease as q increases [13]. Corrections to the amplitude associated with the extrapolation to off-shell energies are also expected to be small at $q > 2$ GeV/c [18].
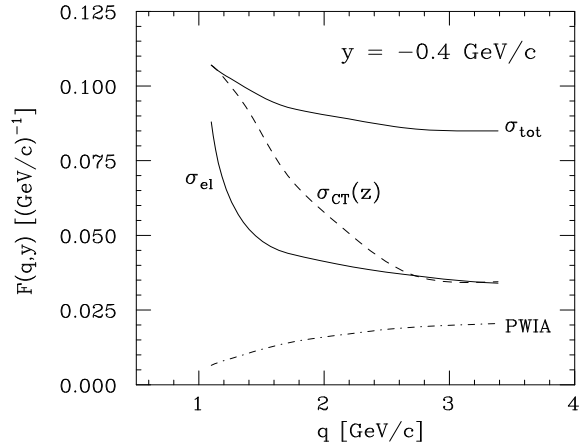

FIG. 3. q-dependence of the nuclear matter scaling function at $y = -$ 0.4 GeV/c. Dash-dot line: PWIA. Solid lines: FSI included using the parametrization of the NN scattering amplitude of refs. [15,16] with the total (σ_{tot}) or elastic (σ_{el}) cross section. Dashed line: color transparency included according to the *quantum diffusion* model of ref. [21].

Modifications of the free-space NN cross section may also originate from the internal structure of the nucleon. It has been suggested [19,20] that elastic scattering on a nucleon at high momentum transfer can only occur if the nucleon is found in the Fock state having the lowest number of constituents, so that the momentum can be most effectively shared among them. This state is very compact, its size being proportional to $1/Q$, and therefore interacts weakly with the nuclear medium. Within this picture a nucleon, after absorbing a large momentum q, travels through nuclear matter experiencing very little attenuation, i.e. exhibits *color transparency* (CT), and evolves back to its standard configuration with a characteristic timescale.

The occurrence of CT is relevant to the analysis of inclusive electron-nucleus scattering at $y < 0$, where elastic scattering is the dominant reaction mechanism, since it leads to a significant quenching of FSI. This effect has been quantitatively investigated in refs. [8–10] using a specific model developed by Farrar *et al.* [21], referred to as *quantum diffusion* model, to describe the time evolution of the NN cross section associated with the onset of CT.

According to ref. [21] the free space NN cross sec-

tion $\sigma_{tot}(q)$ is recovered after travelling a distance $L = 2q/\Delta m^2$, with $\Delta m^2 = 0.7$ GeV^2, whereas at distances $z < L$ the cross section is suppressed and can be written in the form

$$\sigma_{CT}(q, z < L) = \sigma_{tot}(q) \left[\frac{z}{L} + \frac{9 \langle k_t^2 \rangle}{Q^2} \left(1 - \frac{z}{L} \right) \right] \ , \quad (10)$$

where $\langle k_t^2 \rangle^{1/2} \sim 0.35$ GeV/c.

Substitution of the cross section of eq.(10) into the NN scattering amplitude obviously affects the momentum transfer dependence of FSI effects, since the value of the ratio $\sigma_{CT}(q, z)/\sigma_{tot}(q)$ is reduced by a factor $\sim (1/Q^2)$ at $z = 0$ and evolves linearly towards unity within a distance that grows linearly with q.

The results of the calculations of refs. [8–10] show that inclusion of CT effects according to the model of ref. [21], involving no adjustable parameters, greatly improves the agreement between theory and data, allowing for a satisfactory description of the low energy loss tail of the nuclear inclusive cross sections, corresponding to large negative values of y, for $Q^2 > 1.5$ (GeV/c)^2.

The dashed line in fig. 3 shows that CT effects on the nuclear matter $F(q, y)$ are large at $y = -0.4$ GeV/c and $q > 1.5$ GeV/c. The scaling function obtained using the CT cross section of eq.(10) displays a distinct q-dependence, featuring a steep fall in the range $1 < q < 3$ GeV/c, followed by a plateau lying above the PWIA limit. At $q \sim 3$ GeV/c the dashed line and the solid line labelled σ_{el} come very close to each other. This behavior has to be ascribed to the fact that inclusive scattering at large q is mostly sensitive to FSI taking place at distances $z \gtrsim r_c$, $r_c \sim 0.5$ fm being the radius of the repulsive core of the NN interaction [13,18]. In fact, using eq.(10) to model the z dependence of σ_{CT} one finds $\sigma_{CT}(q, z \sim r_c) \sim \sigma_{el}$ at $q \sim 3$ GeV/c.

The analysis described in the present paper shows that, when the rescattering processes are described in terms of free-space NN scattering amplitude, the dominance of inelastic channels gives rise to sizeable FSI effects that show no appreciable q-dependence and persist up to very large values of q. As a consequence, the scaling behaviour of $F(q, y)$ cannot be interpreted as a signature of the onset of the PWIA regime. A similar result was reached in ref. [22], in the context of a study of y-scaling in the nonrelativistic hard-sphere Bose gas.

Inclusion of CT effects leads to a sizeable suppression of FSI effects in the range $1 < q < 4$ GeV/c and to the appearance of a strong q-dependence.

The results of fig. 3 suggest that the analysis of the scaling behavior of the TJNAF data may give a clue to the issue of the possible manifestation of CT in inclusive processes. However, it has to be kept in mind that extracting the experimental scaling function from the measured cross sections requires approximations and assumptions in the treatment of the electron-nucleon ver-tex. Hence, a comprehensive quantitative comparison between theory and data at the level of the nuclear cross sections has to be regarded as a prerequisite to the y-scaling analysis.

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